

4.2 Acceleration

Acceleration

Key Ideas

- The instantaneous acceleration is produced by a change in velocity taken over a very short (infinitesimal) time period. Instantaneous acceleration is a vector in two or three dimensions. It is found by taking the derivative of the velocity function with respect to time
- In three dimensions, acceleration $\vec{a}(t)$ can be written as a vector sum of the one-dimensional accelerations $a_x(t)$, $a_y(t)$, and $a_z(t)$ along the x -, y -, and z -axes.

Learning Objectives

After completing this section, you should be able to:

- calculate the acceleration vector given the velocity function in unit vector notation,
- describe the motion of a particle with a constant acceleration in three dimensions,
- use the one-dimensional motion equations along perpendicular axes to solve a problem in two or three dimensions with a constant acceleration, and
- express the acceleration in unit vector notation.

In [Motion with Constant Acceleration](#), we developed the concept of acceleration in one dimension as the change in a particle's velocity with time. In two and three dimensions, in addition to obtaining the displacement and velocity vectors of an object in motion, we often want to know its acceleration vector at any point in time along its trajectory. This is useful for describing motion when the velocity is not constant, such as when an object is speeding up or slowing down or just changing direction.

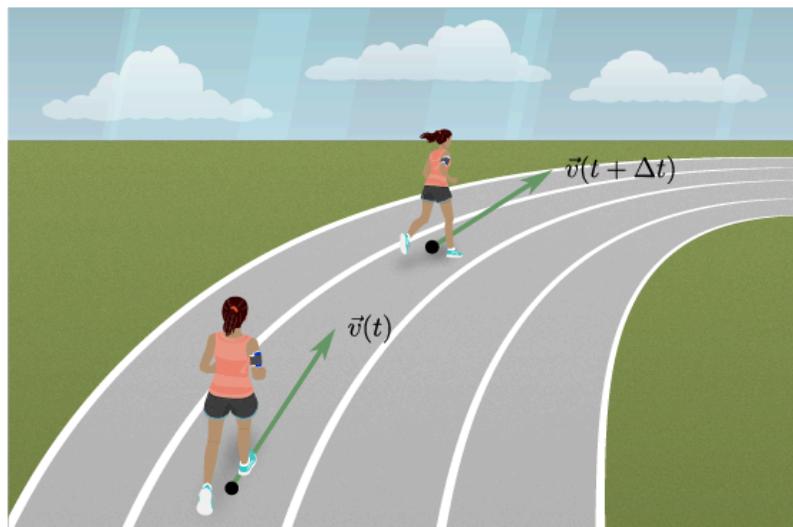


Figure 4.9 The velocity of a point on a runner is represented as the runner moves along an indoor track.

Consider the point on the heel of the running shoe of the runner shown in [Figure 4.9](#). You can imagine that the point is moving in the vertical direction as well as on the track, which is the x - z plane. For simplicity, we show two points on the x - z

plane. Both the magnitude and the direction of the velocity vector can change with time for the runner's heel. The following image simplifies the representation of the velocity vectors for a point at the center of the runner.

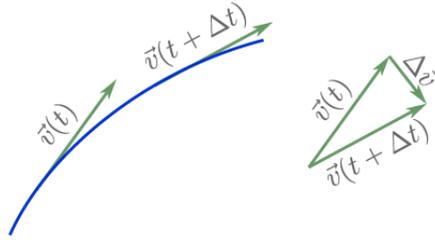


Figure 4.10 (left) The trajectory of the runner is shown, along with two velocity vectors at two points along the trajectory. (right) The velocity vectors are repositioned tail to tail to visualize the change in the velocity of the runner.

From the right side of the figure above, we see that the final velocity is found by adding the change in the velocity to the initial velocity.

$$\vec{v}(t) + \Delta\vec{v} = \vec{v}(t + \Delta t)$$

Then, $\Delta\vec{v} = \vec{v}(t + \Delta t) - \vec{v}(t)$

The change in the velocity is equal to the final velocity minus the initial velocity.

Average Acceleration

During a time interval $\Delta t = t_f - t_i$, the average acceleration is determined using the final and initial velocity vectors.

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

4.7

The SI units for acceleration are m/s^2 .

Instantaneous Acceleration

The instantaneous acceleration is obtained in the limit where the change in time approaches zero, and the two points along the trajectory move increasingly close together.

$$\begin{aligned}\vec{a}(t) &= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt}\end{aligned}\quad \boxed{4.8}$$

The acceleration in terms of components is

$$\vec{a}(t) = \frac{dv_x(t)}{dt} \hat{i} + \frac{dv_y(t)}{dt} \hat{j} + \frac{dv_z(t)}{dt} \hat{k} \quad \boxed{4.9}$$

Also, since the velocity is the derivative of the position function, we can write the acceleration in terms of the second derivative of the position function.

$$\vec{a}(t) = \frac{d^2x(t)}{dt^2} \hat{i} + \frac{d^2y(t)}{dt^2} \hat{j} + \frac{d^2z(t)}{dt^2} \hat{k} \quad \boxed{4.10}$$

When the acceleration is constant, then there is no distinction between the average and the instantaneous acceleration.

Example 4.4

Acceleration of a Swimming Penguin

A zoologist is watching a video clip of a penguin swimming and wishes to determine how fast it is traveling at any given moment in time and how quickly it typically changes its speed. From the clip, she decides that, for the duration of the clip, the position function of the penguin is well-modeled in two dimensions by the function $\vec{r}(t) = bt^2\hat{i} + ct^3\hat{j}$, where $b = 1.1 \text{ m/s}^2$ and $c = 0.3 \text{ m/s}^3$. (a) What is the velocity function? (b) What is the acceleration function? (c) What is the acceleration vector at $t = 2.0\text{s}$? Find its magnitude and direction with respect to the positive x -axis.

Strategize

We will use the definition of velocity as the rate of change of position, [Equation 4.5](#), and the definition of acceleration as the rate of change of velocity, [Equation 4.9](#). In Part (a), we will take the derivative of position to find the instantaneous velocity. In Part (b), we will take the derivative of this velocity function to find the acceleration function. For Part (c), we will substitute the value of $t = 2.0\text{s}$ into the acceleration function.

Develop and Solve

(a) We take the first derivative with respect to time of the position function to find the velocity. The derivative is taken component by component:

$$\vec{v}(t) = 2bt\hat{i} + 3ct^2\hat{j}, \text{ where } b = 1.1 \text{ m/s}^2 \text{ and } c = 0.3 \text{ m/s}^3$$

(b) We take the first derivative with respect to time of the velocity function to find the acceleration. The derivative is taken component by component:

$$\vec{a}(t) = 2b\hat{i} + 6ct\hat{j}, \text{ where } b = 1.1 \text{ m/s}^2 \text{ and } c = 0.3 \text{ m/s}^3$$

(c) Substituting $t = 2.0 \text{ s}$ into the acceleration function gives

$$\begin{aligned}\vec{a}(2.0 \text{ s}) &= 2(1.1 \text{ m/s}^2)\hat{i} + 6(0.3 \text{ m/s}^3)(2.0 \text{ s})\hat{j} \\ &= 2.2 \text{ m/s}^2\hat{i} + 3.6 \text{ m/s}^2\hat{j}\end{aligned}$$

gives the acceleration in unit vector notation. The magnitude of the acceleration is

$$a(2.0 \text{ s}) = \sqrt{(2.2 \text{ m/s}^2)^2 + (3.6 \text{ m/s}^2)^2} = 4.2 \text{ m/s}^2$$

The direction of the acceleration is

$$\phi = \tan^{-1}\left(\frac{3.6 \text{ m/s}^2}{2.2 \text{ m/s}^2}\right) = +58.6^\circ$$

Assess

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